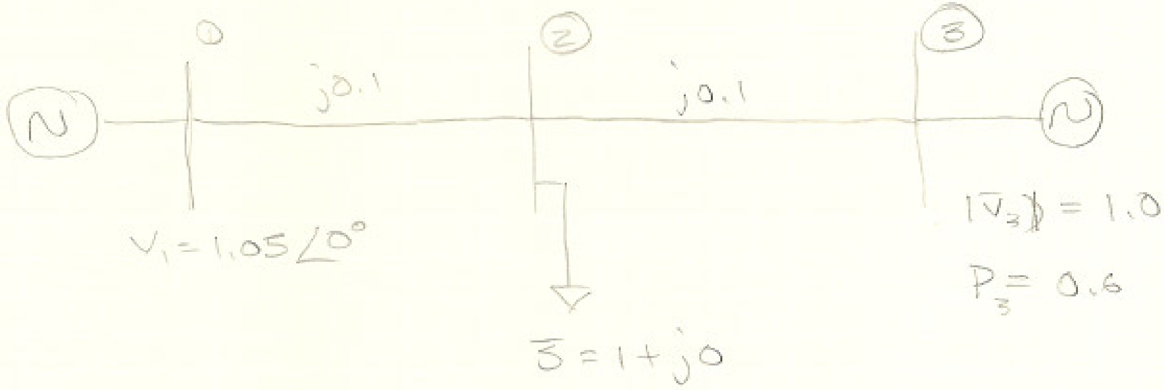
EX: Solve the three bus system using the NR technique



SOL:

$$\boxed{\underline{Y}_{bus} = j\begin{bmatrix} -10 & 10 & 0 \\ 10 & -20 & 10 \\ 0 & 10 & -10 \end{bmatrix}}$$

this problem is simplified by the fact that Y is purely reactive so that:

$$\cos(\delta_k - \delta_j - \theta_{kj}) = \sin(\delta_k - \delta_j)$$

$$\sin(\delta_k - \delta_j - \theta_{kj}) = -\cos(\delta_k - \delta_j)$$

$$P_k = \sum_{j=1}^{n} |\bar{V}_k \bar{Y}_{kj} \bar{V}_j| \cos(\delta_k - \delta_j - \theta_{kj})$$

$$Q_k = \sum_{j=1}^{n} |V_k Y_{kj} V_j| \sin(\delta_k - \delta_j - \theta_{kj})$$

for the NR method we need P_2, Q_2 and P_3 to solve for $|\bar{V}_2|$, δ_2 and δ_3.

$$\begin{aligned} P_2 = {} & |\bar{V}_2||\bar{Y}_{21}||\bar{V}_1| \cos(\delta_2 - \delta_1 - \theta_{21}) \\ & + |V_2 Y_{22} V_2| \cos(\delta_2 - \delta_2 - \theta_{22}) \\ & + |V_2 Y_{23} V_3| \cos(\delta_2 - \delta_3 - \theta_{23}) \end{aligned}$$

$$P_2 = (10)V_2\,(1.05)\sin(\delta_2) + (20)\,V_2^{\,2}\,\cancel{\cos(-\theta_{22})}^{\,0}$$
$$+ |V_2|\,(10)(1)\cos(\delta_2 - \delta_3 - 90^\circ)$$

$$P_2 = \underbrace{10\,V_2\left[1.05\sin\delta_2 + \sin(\delta_2 - \delta_3)\right]}_{\text{calculated.}}$$

$$Q_2 = 10|V_2|\left[2|V_2| - 1.05\cos(\delta_2) - \cos(\delta_2 - \delta_3)\right]$$

$$P_3 = -10|V_2|\sin(\delta_2 - \delta_3)$$

NR:

$$\begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \underline{J} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix}$$

$$\begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \begin{bmatrix} -1 - P_2 \\ 0.6 - P_3 \\ 0 - Q \end{bmatrix}$$

calculated

spec.

$$\underline{J} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_2} \\[2ex] \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} \\[2ex] \dfrac{\partial Q_2}{\partial |V_2|} & \dfrac{\partial Q_2}{\partial |V_2|} & \dfrac{\partial Q_2}{\partial |V_2|} \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = 10\,|V_2|\left[1.05\cos\delta_2 + \cos(\delta_2 - \delta_3)\right]$$

$$\frac{\partial P_2}{\partial \delta_3} = -10\,|V_2|\left[\cos(\delta_2 - \delta_3)\right]$$

$$\frac{\partial P_2}{\partial |V_2|} = 10.5\sin\delta_2 + 10\sin(\delta_2 - \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -10\,|V_2|\cos(\delta_2 - \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_3} = 10\,|V_2|\cos(\delta_2 - \delta_3)$$

$$\frac{\partial P_3}{\partial |V_2|} = -10\sin(\delta_2 - \delta_3)$$

$$\frac{\partial Q}{\partial \delta_2} = 10\,|V_2|\left[1.05\sin\delta_2 + \sin(\delta_2 - \delta_3)\right]$$

$$\frac{\partial Q_2}{\partial \delta_3} = 10\,|V_2|\left[-\sin(\delta_2 - \delta_3)\right]$$

$$\frac{\partial Q_2}{\partial |V_2|} = 10\left[4|V_2| - 1.05\cos(\delta_2) - \cos(\delta_2 - \delta_3)\right]$$

Starting with initial guess

$$|V_2|^{(0)} = 1.0 \text{ pu}$$

$$\delta_2^{(0)} = 0.0$$

$$\delta_3^{(0)} = 0.0$$

$$[J]^{(0)} = \begin{bmatrix} 2.05 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1.95 \end{bmatrix}$$

$$\begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \begin{bmatrix} -1 \\ 0.6 \\ 0.5 \end{bmatrix}$$

$$\Delta \delta_2^{(0)} = -0.038095 \text{ rad}$$

$$\delta^{(1)} = 0 - 0.038093 = -0.03895 \text{ rad.}$$

$$\Delta \delta_3^{(0)} = 0.021905$$

$$\delta_3^{(1)} = 0 + 0.021905 = 0.021905 \text{ rad.}$$

$$\Delta |V_2|^{(0)} = 0.025641$$

$$|V_2|^{(1)} = 1 + 0.025641 = 1.02564 \text{ PU}$$

using the above values

$$P_2^{(1)} = 1.02517$$

$$\Delta P_2^{(1)} = 0.02517$$

$$Q^{(1)} = 0.0394187$$

$$\Delta Q^{(1)} = -0.0394187$$

$$P_3^{(1)} = 0.615015$$

$$\Delta P_3^{(1)} = -0.015015$$

$$\begin{bmatrix} 0.02517 \\ -0.015017 \\ -0.039419 \end{bmatrix} = \begin{bmatrix} 2.04744 & -0.998201 & -0.05954 \\ -0.988201 & 0.998201 & 0.059964 \\ -0.099954 & 0.059964 & 2.05512 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix}$$

$$\Delta\delta_2^{(1)} = 8.76 \times 10^{-4}$$

$$\delta_2^{(1)} = -0.037219$$

$$\Delta\delta_3^{(1)} = -4.83 \times 10^{-4}$$

$$\delta_3^{(1)} = 0.021421$$

$$\Delta|V_2| = -1.8135 \times 10^{-3}$$

$$|V_2| = 1.02378$$

Let us say that we are satisfied with the solution using the above values, we get.

$$\boxed{\begin{aligned} \bar{S}_1 &= P_1 + jQ = 0.40017 + j0.282744 \\ \bar{S}_2 &= -1.600004 + j8.49 \times 10^{-5} \\ \bar{S}_3 &= 0.600025 - j0.22021 \end{aligned}}$$

note: A third iteration would give us a error of 10^{-10}